Towards a Reproducible, Verifiable Parallel Graph Analysis Benchmark

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What is a benchmark?

Merriam-Webster: A standardized problem or test that serves as a basis for evaluation or comparison (as of computer system performance)
Aesop’s Benchmark
The Tortoise and the Hare
Aesop’s Benchmark

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Aesop’s Benchmark
The Tortoise and the Hare

Moral of the story: competitors must complete the same task to be compared fairly
Computer Benchmark Learnings

• Many mistakes are unintentional:
  – Algorithmic “improvements”
  – Shortcuts: only test some results
  – Unwarranted compiler overrides

• Some mistakes are (semi-)intentional
• Most mistakes are eliminated with a good verification test—debugging aid!

The code in this if test never gets executed anyway

I know this vectorizes, trust me

My code does not have races

It’s equally random each time, so each run with races is qualitatively the same

My code does not have races
What makes a good verification test?

• Frugal; test should not be too onerous
  – Time(test) <= Time(problem)
  – Memory/disk(test) <= Memory/disk(problem)

• No false negatives; test should not be too tight
  – Legitimate algorithmic variation should pass
  – Don’t demand bitwise identical floating point computations

• No false positives; test should not be too lax
  – Mr. Bijk’s zero current test
  – HPCC RandomAccess table test
  – NAS Parallel Benchmarks Multi-Grid convergence test
  – S3D reaction rate test
  – SSCA2: Problem verified different from problem measured
Scalable Synthetic Compact Application 2 (SSCA2)

• Construct small-world directed graph
  o Use R-MAT for edge generation
  o Reject self edges (trivial) and duplicate edges (hardish)
  o Permute vertex indices to remove first-quadrant bias
  o Assign random, uniformly distributed weights to edges

• Compute Betweenness Centrality (BC) for all vertices
  ❑ Use set of vertices as sources for Breadth First Search (BFS) in shortest path computations
  ❑ Skip graph edges whose weight is divisible by 8
  ❑ Measure Traversed Edges Per Second (TEPS)
What is betweenness centrality?

Definitions:
- \( s, v, t \in V \) (set of vertices of the graph)
- \( \sigma_{st} \): # all shortest paths from \( s \) (source) to \( t \)
- \( \sigma_{st}(v) \): # shortest paths from \( s \) (source) to \( t \), passing through \( v \)
- dependency \( \delta_{st}(v) = \sigma_{st}(v)/\sigma_{st} \)
- Betweenness centrality: \( BC(v) = \sum_{s \neq v \neq t} \delta_{st}(v) \)

Efficient recent implementations use BFS. Typically two distinct steps:
1. Do BFS from subset of source vertices \( s \); assign to each other vertex \( v \) the distance and shortest paths count from \( s \)
2. Reverse traversal, accumulate \( \delta_{st}(v) \) (step 1) in BC, using simple recursive relation
BC $\approx$ vertex degree?

High vertex degree, high-ish BC
BC $\approx$ vertex degree?

“hub” vertex: high BC, low vertex degree
BC ≈ vertex degree?

Pearson correlation coefficient of BC versus vertex degree for SSCA2
BC \approx vertex degree?

Pearson correlation coefficient of BC versus vertex degree for SSCA2

Complete graph: everybody has high vertex degree, BC \equiv 0
Scalable Synthetic Compact Application 2 (SSCA2)

• Don’t verify, because parallel implementation of
  1. R-MAT gives different results for different numbers of threads (no races)
  2. Permutation of vertex indices gives different results for different numbers of threads, and has races
  3. Selection of source vertices gives different results for different numbers of threads, and has races
  4. Filtering of edge duplicates/weights assignment has races

• Don’t verify, because it can’t be done for graph sizes that have never been generated before
1. R-MAT gives different results for different numbers of threads

• Each thread uses independent stream of (Pseudo) Random Numbers (PRNs) to create edges
• Fix: use single stream of PRNs, and distribute sub-streams evenly among threads
• Requires: jump ahead in single stream in $O(\log N)$ time for jump N
  – Easy for multiplicative Linear Congruential Generators: $x_k = (a \times x_{k-1}) \mod m$, $x_0 = s$, but usually short period (e.g. NAS Parallel Benchmarks: $2^{46}$)
  – Harder for mixed LCGs with longer period ($2^{64}$): $x_k = (a \times x_{k-1} + c) \mod m$, but can be done

\[
x_k = \left( a^k s + c \left( \sum_{i=0}^{k-1} a^i \right) \right) \mod m = \left( a^k s + c\frac{a^k - 1}{a - 1} \right) \mod m
\]

• RMAT needs $5 \times n \times \log(n)$ PRNs to create $n$ edges; LCG with period $2^{64}$ can generate $2^{55}$ edges before duplication starts.
• Current max for Graph500:\footnote{source: www.graph500.org, June 2011}: $n=2^{42}$ (Intrepid, IBM Blue Gene/P; 32,768 nodes; 131,072 cores). Storage: 70TB.
Comparing vertex degree distributions

Vertex degree distribution
Scale=26, 32 threads

#occurrences

0 1 10 100 1000 10000 100000 1000000

0 500 1000 1500 2000 2500 3000

Single stream
Original
Comparing vertex degree distributions

Vertex degree distribution
Scale=26, 32 threads

#occurrences

Vertex degree
2. Vertex permutation not reproducible

• Original implementation for n vertices:
  – Each thread generates \((n/n\text{threads})\) PRNs \textit{independently}
  – Each pair \((i,\text{PRN}(i))\) defines vertex swap pair
  – If any other thread tries to swap overlapping pair simultaneously, abandon swap (locks); race, even if no swap ever abandoned (swaps don’t commute)

• Fix:
  – Explicit permutation function \(F\), mapping vertex \(i\) to vertex \(F(i)\)
  – \(F(i) = \text{bit-reverse}(i)\) XOR pseudo-random mask

• The good, the bad:
  – Deterministic, cheap to compute (vectorizable), no locks (OpenMP) or communication (MPI), good scattering.
  – Can be reverse engineered. More regular than random swaps
Permutation functions

Case 1
Case 2
Case 3
Case 4
3. Source vertex selection not reproducible

- Uses permutation of vertex indices
- Fix:
  - Explicit permutation function $F$, mapping vertex $i$ to vertex $F(i)$
Intermezzo

After introduction of single stream of PRNs in R-MAT and of explicit vertex permutation function:

• Verification value $\Sigma BC$ identical from run to run, for different numbers of threads, if duplicate edges allowed

• $\Sigma BC$ different from run to run if duplicate edge filtering turned on

• Conclusion: race in duplicate edge filtering
4. Edge weight assignment has races

- BFS ignores edges if: weight \((\text{mod} \ 8) = 0\)
- Weights assigned uniformly, randomly, using PRNs
- PRNs based on edge sequence number (and thread ID)
- If duplicate edges filtered:
  - Any specific edge inserted only once
  - First thread to arrive inserts edge in its local edge list
  - Local edge lists merged through concatenation
- Result: edge weights experience race
4. Edge weight assignment has races

• Fix 1: sort edges before assigning weights
  – $O(n\log(n))$ algorithmic complexity
• Fix 2: remove duplicates afterwards, keeping only first occurrence
  – need to repeat until enough unique edges added
  – $O(n\log(n))$ algorithmic complexity
  – higher programming complexity
• Fix 3: 
  – employ physical parameters (end points) to assign weight
  – Simplest solution: seed = start_id+end_id; apply LCG once
  – Number of ignored edges very close to required $1/8^{th}$.
  – Uniform?
Experimental results, scale=24, 7 runs

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Allow duplicate edges
Conclusions I

SSCA2:
• Can be made reproducible *without reading graph from file*
• Can be made verifiable
  – Anyone running benchmark must:
    • match prior results, or
    • publish newly obtained result
• Has been used with success after modifications
  – Very quick debugging of performance optimizations
  – Original runtime variations \(\approx\) optimization gains
• Has some open issues
  – Vertex permutation sufficiently random?
  – Weight assignment sufficiently uniform?
Graph500

- Does BFS like SSCA2, but no Betweenness Centrality; Single-Source Shortest Paths being considered
- PRNG: Multiple Recursive Generator for random number generation $x_k = (a_1 * x_{k-1} + ... + a_n * x_{k-n}) \mod m$ with jump ahead
- Permutation:
  - Parallel Random Sort based permutation (v1.x)
  - Suggested bit reversal in October 2010
  - Moved to bit reversal-based permutations for vertex scrambling in April 2011 (v2.x)
- No edge weights assignment or filtering

\(^\dagger\)graph not reproducible; by design or because of races
Benchmark + verification time

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Conclusions II

Graph500:

• Fixes reproducibility ills of SSCA2
• Should spend less time verifying solution
• Should be augmented with more demanding kernel à la BC
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